A SIMPLE ANALYSIS OF THE NON-UNIFORM FIELD EFFECTS ON THE GUNN DEVICES

TAPAN BHATTACHARYA

ELECTRONICS DIVISION, COMPONENTS SECTION, ATOMIC ENERGY ESTABLISHMENT TROMBAY, BOMBAY 74 (AS)

(*Received December* 6, 1966)

ABSTRACT. The effects of having a non-uniform field distribution on the efficiency of Gunn devices are studied from a device designer's viewpoint. Two means are suggested to achieve such field distributions : (i) resistivity gradient, and (ii) non-planar electrode geometry. Based on simplified mathematical models, both the cases are analysed for the output power and efficiency. The performance of these proposed devices is compared with the usual uniform-field Gunn devices, and the advantages, disadvantages and practicability of such devices discussed. It is shown that in certain ranges of the high-field domain width and the microwave frequency, the non-uniform field devices may be more efficient than their counterpart.

INTRODUCTION

There is much recent interest in the Gunn-effect oscillators (Gunn, 1964a, b) made of two-valley compound semiconductors. The differential-negative-conductance model, independently proposed by Ridley and Watkins (1961) and Hilsum (1962), predicts current instabilities in a two-valley semiconductor such as GaAs or InP when the applied electric field exceeds a threshold value. Ridley (1963) predicted the formation of a high-field domain and a low-field domain in the bulk once the applied field is in the region of negative differential conductivity. During and after its formation the high-field domain (HFD) propagates through the length of the sample with roughly the low-field electron drift velocity. The periodic nucleation and propagation of this HFD give rise to the microwave power output. The frequency of oscillation is given by the inverse of the domain transit time. McCumber and Chynoweth (1966) and Kroemer (1966) have shown that normal statistical fluctuations of the donor density in the semiconductor give rise to a dipole-domain formation. While the width of a pure accumulation-layer domain is directly proportional to the length of the sample (Ridley, 1963), no such simple relation exists in the case of a more practical dipole-layer domain, and it seems likely that the length of the sample does not have much influence on the width of a pure dipole-layer domain (Copeland, 1966a; Chynoweth, 1966). This is also borne out by the observation that coherent sinusoidal oscillations are obtained in shorter samples as compared to spiked oscillations in relatively longer samples (Foyt and McWhorter, 1966). However, the width of both types of domains is influenced strongly by the applied field, the width increasing with increasing applied field (Allen *et al.*, 1966;Butcher *et al.*, 1966; Copeland, 1966a). Heeks (1966) observed that the field in the HFD saturates to around 75 KV/cm. and any excess applied voltage is absorbed by the domain by increasing in its width.

Gunn-effect oscillators have been successfully fabricated in the kilo-mega cycle frequency range with peak pulsed power output of about 100 watts (Dow *et al.*, 1966) and a few tens of milliwatts in the CW operation (Hakki and Knight, 1966). The efficiency of such devices is typically 3 to 10 percent, which compares well with the 2 percent efficiency of the present-day low-power single-cavity reflex klystrons. There have been some efforts made to calculate the efficiency of such devices (Hilsum, 1965; Copeland, 1966b). However, such calculations become extremely complicated because of the inherent non-linear nature of the device and computer solutions are usually resorted to. Hilsum (1965) considered an accumulation-layer domain equivalent to a charged capacitor and by considering the energy stored in this capacitor, arrived at a simple expression for the efficiency of the device.

In a series of experiments Heeks and coworkers (Heeks *et al.*, 1965; Heeks, 1966) demonstrated Gunn's original observation that only the nucleation of the HFD requires the applied field to be higher than the threshold field, but a much lower field is necessary to maintain the propagation of the HFD along the length of the sample. This demonstration immediately suggests that the efficiency of the Gunn device could possibly be increased by keeping the maintaining field low after the complete formation of the HFD, because then the Joule heat loss will be less. A further step ahead would be to convert the time variation of the applied field (Heeks, 1966) to a spatial variation in the sample, thus introducing a non-uniform field distribution. The purpose of this paper is to analyse this situation from a macroscopic viewpoint with the assumption of simplified mathematical models. Two different means are suggested to achieve such non-uniform field distributions and both the cases are analysed to obtain expressions for the efficiency. Comparison is made with the existing uniform-field Gunn devices and the advantages, disadvantages and the practicability of the proposed techniques are discussed.

RESISTIVITY GRADIENT

Kroemer (1966) suggested that intentional resistivity gradient may be used in the Gunn devices either to force the formation of a single depletion layer rather than a multiple of such layers, or, with a reverse gradient, to force the suppression of such depletion layers and formation of pure accumulation layers. Das and Maron (1966) analysed this situation and showed the desirability of having a non-uniform carrier distribution. We consider here an exponential distribution of the electron density:

$$n(z) = n_0 e^{(az/l)} \quad \dots \quad (1)$$

where z is the distance along the sample of length l, and α is a constant. With an applied voltage V across the sample, the electric field $F(z)$ is given by :

$$F(z) = \frac{\alpha V e^{-(\alpha z/l)}}{(1-e^{-\alpha})} \qquad \dots \quad (2)$$

where the built-in thermal field (of the order of $\frac{kT}{lq}\,\alpha$) is neglected. The low domain field F_L required to maintain the propagation of the HFD may be written as :

$$F_L = (F_T/m) \qquad \dots \quad (3)$$

where F is the threshold field and m is a constant greater than 1. For successful nucleation and propagation of the HFD we demand that :

$$\left.\begin{array}{l} F(z) = F_L \,;\; z = l \\ F(z) \geqq F_T \,;\quad z \leqq z_0 \end{array}\right\} \qquad \dots \quad (4)$$

Equations (2), (3) and (4) give :

$$V = (F_T l/\alpha m)(e^{\alpha}-1), \qquad \dots \quad (5)$$

$$F(z) = (F_T/m)e^{\alpha}e^{-(\alpha z/l)}, \qquad \dots \quad (6)$$

$$z_0 = l(1-\ln m/\alpha). \qquad \dots \quad (7)$$

The field distribution, as outlined above, is shown schematically in Fig. 1(a). Kroemer (1966) considered the formation mechanism of both an accumulation-layer and a dipole-layer HFD due to the statistical fluctuations in the carrier density. Applying Kroemer's argument in this case, with the existing non-uniform field, it is easily seen that the domain formation will be one of a dipole layer,

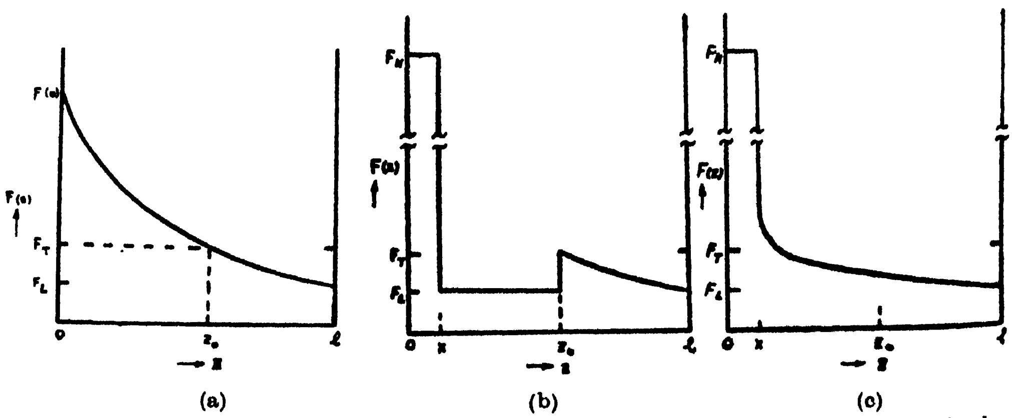

Fig. 1. Schematic diagram showing (a) the field distribution in the sample (Eq. (6)) immediately after the application of the external bias, (b) the assumed transient distribution at the time of the formation of the high-field domain, and (c) the final steady state distribution after the complete domain formation.

because the upstream field is higher than the down-stream field. We proceed with our calculations keeping in mind the salient features of a dipole-layer doman discussed in the Introduction.

In order to arrive at an expression for the efficiency of such a device, we make the following assumptions :

(i) The HFD is flat-topped, and the domain voltage is given reasonably accurately by :

$$V_H = F_H x \qquad \dots \quad (8)$$

where F_H, the field in the HFD, saturates to a value given by :

$$F_H = (F_T/s) \qquad \dots \quad (9)$$

where s is a constant less than 1 (Heeks, 1966). Any excess applied voltage is absorbed in the domain by increasing its width x.

(ii) The formation of the HFD takes place according to the Figs. 1(b) and 1(c). The potential drop in the region $0 < z < z_0$ is only responsible for the formation of the HFD (Fig. 1(b)), but after its formation, the potential gets redistributed as shown in Fig. 1(c).

(iii) The HFD is equivalent to a charged capacitor C, whose stored energy propagates through the length of the sample in time

$$\tau = (l/v) \qquad \dots \quad (10)$$

to deliver the microwave power (Hilsum, 1965). Here the low-field electron drift velocity v is given by :

$$V = F_L \mu = F_H \mu' \qquad \dots \quad (11)$$

where μ and μ' are the mobilities of the electrons in the lower and upper valleys respectively.

The weakest of the assumptions is (i) above. This assumption necessarily means that the wall thickness of the accumulation and depletion layers is negligible compared to the total width of the HFD. This, in turn, restricts the carrier density to rather high values (Allen *et al.*, 1966; Butcher *et al.*, 1966; Copeland, 1966*a*). Assumption (ii) is the basis of the mathematical model we use for analysing the situation, and in the absense of any contradictory evidence we are willing to base our calculations on this model.

Under these simplifying assumptions, the average power output of the device for a time large compared to τ is given by :

$$P_0 = \frac{\frac{1}{2} C V_H^2}{(l/v)} = \frac{K F_T^2 y \mu}{8\pi s^2 m} \qquad \dots \quad (12)$$

where K is the dielectric constant of the material and the relative domain width is given by :

$$y \equiv (x/l). \qquad \dots \quad (13)$$

where z is the distance along the sample of length l, and α is a constant. With an applied voltage V across the sample, the electric field $F(z)$ is given by :

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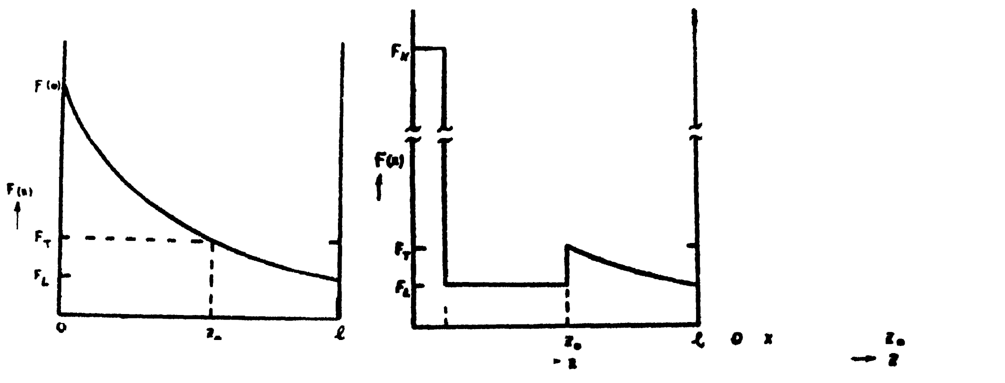

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Under these simplifying assumptions, the average power output of the device for a time large compared to τ is given by :

$$P_0 = \frac{\frac{1}{2}CV_H^2}{(l/v)} = \frac{KF_T^3 y\mu}{8\pi s^2 m} \qquad \dots \quad (12)$$

where K is the dielectric constant of the material and the relative domain width is given by :

$$y \equiv (x/l). \qquad \dots \quad (13)$$

Equation (12) is derived with the help of Eqs. (3), (8), (9) and (11). Since the carriers in the HFD are in the upper valley of the conduction band, the resistance of this domain is approximately given by :

$$R_H = \int_0^x \frac{dz}{q\mu' n(z)} = \frac{l(1-e^{-\alpha y})}{\alpha n_0 q\mu'} \qquad \ldots \quad (14)$$

where q is the electronic charge. For low values of the domain width, this resistance obviously reduces to :

$$R_H \approx (x/n_0 q\mu'). \qquad \ldots \quad (15)$$

The resistance of the low field region is similarly given by .

$$R_L = \int_x^l \frac{dz}{q\mu n(z)} = \frac{l}{\alpha n_0 q\mu}(e^{-\alpha y} - e^{-\alpha}) \qquad \ldots \quad (16)$$

The potential drop in the low-field region is given from Eqs. (5), (8) and (9) by ·

$$V_L = V - V_H$$

$$F_T l \quad [s(e^{\alpha}-1)-\alpha m y]. \qquad (17)$$

The power input to the device under these conditions is given by

$$P = \frac{V^2{}_H}{R_H} + \frac{V^2{}_L}{R_L} \qquad (18)$$

and with the aid of Eqs. (8), (9), (14), (16) and (17) this becomes :

$$P = \frac{F_T{}^2 q n_0 \mu l}{\alpha m^2 s^2}\left[ms(1-e^{-\alpha y}) + \frac{\{s(e^{\alpha}-1) - \alpha m y\}^2}{\{e^{-\alpha y} - e^{-\alpha}\}} \right] \qquad (19)$$

From Eqs. (12) and (19) the overall efficiency of the device is :

$$\eta = \frac{K F_T}{8\pi q n_0 l}\left[\alpha m y \left\{ ms(1-e^{-\alpha y}) + \frac{[s(e^{\alpha}-1)-\alpha m y]^2}{[e^{-\alpha y} - e^{-\alpha}]} \right\}^{-1} \right] \qquad (20)$$

The efficiency thus calculated is plotted in Fig. 2 as a function of the relative domain width y with α as a parameter. The following values, typical of n-GaAs, are used for this purpose :

$$\begin{aligned} m &= 2 \\ s &= 0.043 \\ F_T &= 3.2 \times 10^3 \text{ volt/cm.} \\ K &= 1.1 \times 10^{-10} \text{ farad/meter,} \\ \mu &= 5000 \text{ cm}^2\text{/volt-sec.} \end{aligned} \qquad \ldots \quad (21)$$

Also plotted in this figure is the efficiency of a uniform-field device (the curve marked $\alpha = 0$), the expression for which was derived by Hilsum (1965). The uniform carrier density assumed for this plot is equal to n_0. Figure 2 clearly shows that for large values of y and relatively low α, the efficiency of the non-uniform field device, as described above, is larger than that for the uniform field device. The value of α cannot be much less than unity, because Eqs. (4) and (7) have to be simultaneously satisfied. Decreasing α automatically reduces z_0, and since y cannot be larger than (z_0/l), so the advantage obtained by decreasing α is lost. The minimum value of α, as seen from Eq. (7), is approximately 0.69, in which

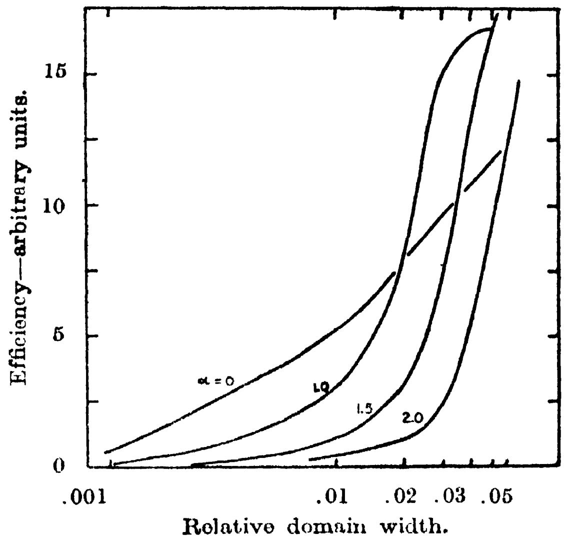


Fig. 2. The computed efficiency of the Gunn device plotted as a function of the relative domain width, for the uniform-field ($\alpha = 0$) and the non-uniform field ($\alpha > 0$) Gunn devices.

case there will not be any stable domain formation and oscillation. From known observations (Hilsum, 1965; Heeks, 1966) it can be safely assumed that the relative domain width y lies somewhere between 2 to 4 percent. With this range of values for y, Fig. 2 shows that having a non-uniform carrier distribution with $\alpha = 1$ does give a more efficient device than the uniform device of same length and having a carrier density of n_0. It may not be possible to obtain such low gradient of carrier density ($\alpha = 1$) with the techniques of indiffusion. However, the usual donors in GaAs, such as Se and Te, have rather low diffusion constants, and the technique of out-diffusion could be employed to obtain the required carrier-density gradient.

NON-PLANAR ELECTRODE GEOMETRY

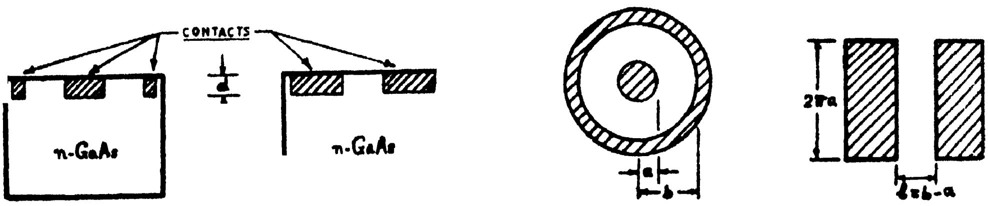
The second method to achieve the non-uniform field distribution in the Gunn devices is to use non-planar electrode geometry. We will consider here the case of concentric cylindrical electrodes as illustrated in Fig. 3. Such devices could be fabricated by selective surface plating of the contact electrodes by the usual photolithographic techniques and then alloying. Similar surface-oriented Gunn devices with planar electrodes have been successfully used (Foxell *et al.*, 1965). The following analysis is made by adopting all the assumptions made in the previous case except (ii). With the symbols defined in Fig. 3, the average power output of the cylindrical device is given from Eqs. (8), (9), (10) and (11) as :

$$P_{OC} = \frac{\frac{1}{2} C_C V_H^2}{(b-a)/F_L\mu}$$

$$= \frac{KdF_L\mu F_2^H x^2_C}{4(b-a)\, l_n\, (1+x_C/a)}$$

$$= \frac{KdF_3^T x_C \mu}{4(b-a)s^2 m\, l_n\, (1+x_C/a)} \qquad \ldots \quad (22)$$

where the subscript c denotes the cylindrical case. Instead of calculating the power input to the device and the efficiency, as was done earlier, we will calculate the power output in the case of a planar-electrode device (case "p" in Fig. 3), and



Case : "c" Case : "p"

Fig. 3. Electrode configurations for the cylindrical (case "c") and the planar (case "p") devices.

assuming the same power input to both the devices, compare the output powers and hence the efficiencies. The latter case has been treated by Hilsum (1965), and in our notations the output power of the planar device is given by :

$$P_{OP} = \frac{\frac{1}{2} C_P V_2^H}{(b-a)/F_L\mu}$$

$$= \frac{KdaF_3^T x_P \mu}{4s^2 m(b-a)}\,. \qquad \ldots \quad (23)$$

As a basis for the comparison, we have assumed here that the depth of alloying d(see Fig. 3) is the same in both the cases and the width of the planar electrode is $(2\pi a)$. With the same input power to both the devices, the ratio of the respective efficiencies is given from Eqns. (22) and (23) as :

$$\frac{\eta_C}{\eta_P} = \frac{P_{OC}}{P_{OP}}$$

$$= \left(\frac{x_C}{x_P}\right) \frac{(x_C/a)}{ln\,(1+x_C/a)} . \qquad \ldots \quad (24)$$

Since for any positive x_C :

$$\left(\frac{x_C}{a}\right) > ln \left\{ 1+ \left(\frac{x_C}{a}\right) \right\}, \qquad \ldots \quad 25)$$

hence from Eq. (24)

$$\frac{\eta_C}{\eta_P} > \frac{x_C}{x_P} . \qquad \ldots \quad (26)$$

So, if $x_C > x_P$, then η_C will always be larger than η_P.

The field variation in the case "c" follows an inverse radial-distance law. With the boundary conditions enumerated in Eqs. (4) above (with the radial distance r substituted for z), we will have an identical situation here as in the previous case. Following the arguments given there and in the Introduction, it is possible that x_C will be larger than x_P, which will ensure that η_C is always larger than η_P. This was not unexpected, because the capacitance, and hence the energy stored in it with the same maximum field F_H, is larger in the case of cylindrical electrodes than that in the planar case with the same cross sectional area. Apart from this increase in the capacitance because of the geometry, there could be further increase in the efficiency of the cylindrical device because of the possible increase in the domain width due to the non-uniform field distribution.

Referring to Eqs. (25) and (26), it is noted that larger the value of (x_C/a), larger will be the increase in η_C compared to η_P. With the present-day photolithographic techniques a could be as small as 3-5 microns, and with x_C presumably in the same range, there could be a substantial increase in the efficiency of the cylindrical device as compared to that in the planar device. Of course, the reduction in a will necessarily reduce the cross sectional area and hence the output power. Another interesting observation that can be made here is that the HFD travels radially through the active length $l = (b-a)$, and unlike the linear flow its width is reduced during its propagation. This will, therefore, tend to give rise to spiked waveform rather than sinusoidal ones (Foyt and McWhorter, 1966). This effect could conceivably be overcome by keeping the active length $(b-a)$ reasonably small. This reduction in the active length, which is one of the great advantages of the surface-oriented devices (Foxell *et al.*, 1965), helps generating higher frequency.

ACKNOWLEDGMENT

The author wishes to thank M. B. Khambaty for introducing him to this field

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